Disorder effect of resonant spin Hall effect in a tilted magnetic field

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We study the disorder effect of resonant spin Hall effect in a two-dimensional electron system with Rashba coupling in the presence of a tilted magnetic field. The competition between the Rashba coupling and the Zeeman coupling leads to the energy crossing of the Landau levels, which gives rise to the resonant spin Hall effect. Utilizing the Streda's formula within the self-consistent Born approximation, we find that the impurity scattering broadens the energy levels and the resonant spin Hall conductance exhibits a double peak around the resonant point, which is recovered in an applied tilted magnetic field.

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I. INTRODUCTION

Spin-orbit couplings open a route to control quantum electron spin by electric means. One of the efficient methods to inject or generate electron spin in nonmagnetic semiconductors is the spin Hall effect, in which an electric current or an electric field may induce a transverse spin current in the systems with strong spin-orbit couplings. Early theories[1,2] proposed that the spin current is caused by asymmetric scattering of electrons with spin up and down in impurity potentials, named as extrinsic spin Hall effect. In recent years it was demonstrated that the spin-orbit coupling in the electron bands can also lead to an intrinsic spin Hall effect in either p-doped or n-doped semiconductors.[3,4] Both extrinsic and intrinsic spin Hall effects were confirmed experimentally in various systems.[5–8]

A two-dimensional electron gas (2DEG) with a Rashba coupling was proposed to exhibit an intrinsic spin Hall effect.[4,9] The spin-orbit coupling in 2DEG modifies the electron band structure and may lead to interesting magnetotransport properties, such as the beating phenomenon in the Shubnikov-de Haas oscillation.[10,11] When the system is subjected to an external magnetic field, the Zeeman splitting will also change the spin-dependent electron bands. The interplay of the spin-orbit coupling and the Zeeman coupling produces the crossing of electron energy levels. Based on this property, it was proposed that a tiny electric field may remove the additional degeneracy of energy levels and produces a finite spin current if the Fermi surface sweeps across the crossing point of energy levels. As a result, there exhibits a divergent spin Hall conductance.[12–15] This resonant spin Hall effect was also discussed in p-doped systems in a magnetic field.[16,17]

However, impurities in the system make the issue more subtle.[18,19] The vertex correction in the self-energy turns out to cancel the spin Hall conductance even in a weak disorder limit in 2DEG with linear Rashba coupling while the spin Hall conductance survives in p-doped Luttinger model and the systems with cubic spin-orbit couplings.[20–25] The disorder effect strongly depends on the symmetry of the spin-orbit coupling and the dispersion. Now whether the resonant spin Hall effect can survive in a finite density of impurities becomes an issue to be answered. This is the motivation of the present work.

Here we present a full investigation on the disorder effect of resonant spin Hall effect in 2DEG with the Rashba coupling in a tilted magnetic field. The impurity effect is considered by the self-consistent Born approximation (SCBA) and the vertex correction in the ladder approximation. We found the impurity effect will suppress the resonant spin Hall conductance at the resonant point and produce a double-peak structure of the spin Hall conductance around the point. A tilted magnetic field is applied to enhance the effective Zeeman splitting[26–30] and to recover the effect when the energy-level splitting is larger than the energy broadening by impurities

II. GENERAL FORMALISM

A. 2DEG in a tilted field

We consider a 2DEG in the x-y plane with the Rashba spin-orbit interaction in a tilted magnetic field. The perpendicular component of the tilted field is $B_\perp$ and the in-plane component is chosen to be along the x direction $B_\perp \tan\theta$, where θ is the angle between the field and the z direction. We take the Landau gauge for the vector potential of the field $\vec{B}=(B_\perp \tan\theta, 0, -B_\perp)$. The total Hamiltonian including the Zeeman energy is given by

$$H_0 = \frac{1}{2m}[(p_x + eB_\perp y)^2 + p_y^2] + \frac{\lambda}{\hbar}[(p_x + eB_\perp y)\sigma_y - p_y\sigma_x] - \frac{1}{2}g_s\mu_B B_\perp \sigma_z + \frac{1}{2}g_s\mu_B B_\perp \tan\theta\sigma_x, \tag{1}$$

where $\mathbf{p}=-i\hbar\nabla$, $m, -e, g_s$ are the electron's effective mass, charge, and Lande g factor, respectively. μ_B is the Bohr magneton, λ is the strength of Rashba spin-orbit coupling, and σ_i are the Pauli matrices. We take a periodic boundary condition along the x direction, hence the momentum $p_x=\hbar k$ is a good quantum number.

An analytical solution can be obtained in the case of $\theta=0$.[12,31,32] There were some studies on spin transport based on the solution.[13,33,34] Inclusion of the tilted field makes the problem much more complicated and an analytical solution is not available at present. In the following approach, we choose the energy eigenstates for the system without the spin-orbit coupling ($\lambda=0$) and $\theta=0$ as a set of basis,

$$|nk\sigma\rangle = \frac{1}{\sqrt{L_x}} e^{ikx}\phi_n(y+kl_b^2)|\sigma\rangle, \tag{2}$$

where the magnetic length $l_b=\sqrt{\hbar/eB_\perp}$, the spin index $\sigma=\uparrow,\downarrow$, $L_{x(y)}$ is the length of the 2DEG, $\phi_n(y)$ is the eigenstate of the n^{th} energy level of a linear oscillator with the frequency $\omega=eB_\perp/m$,[35] and $|\sigma\rangle$ is the eigenstate of spin σ_z.

When the tilt angle $\theta=0$, the system can be solved exactly.[12,31] The eigenvalues of H_0 are given by

$$\epsilon_{ns} = \hbar\omega\left(n + \frac{s}{2}\sqrt{(1-g)^2 + 8n\eta^2}\right), \tag{3}$$

where $\eta=\lambda m l_b/\hbar^2$ and $g=g_s m/2m_e$ with m_e the mass of a free electron, $s=1$ for $n=0$, and $s=\pm 1$ for $n\geq 1$. The states Φ_{nks} have a degeneracy $N_\phi=L_xL_yeB/h$, corresponding to N_ϕ values of k. The eigenstate has the form

$$\Phi_{nks} = \cos\,\theta_{ns}|n,k,\uparrow\rangle + i\,\sin\,\theta_{ns}|n-1,k,\downarrow\rangle, \tag{4}$$

where $\theta_{01}=0$, and for $n\geq 1$, $\theta_{ns}=\arctan(-u_n+s\sqrt{1+u_n^2})$ with $u_n=(1-g)/\sqrt{8n}\eta$. One of the features of the solution is the crossing of the energy levels as functions of the magnetic field, which is caused by the competition between the spin-orbit coupling and Zeeman energy splitting. For the two levels ϵ_{n1} and $\epsilon_{n+1,-1}$, the condition for the crossing is determined by[12]

$$\sqrt{(1-g)^2 + 8n\eta^2} + \sqrt{(1-g)^2 + 8(n+1)\eta^2} = 2. \tag{5}$$

This point is called the resonant point for resonant spin Hall effect.

This additional degeneracy due to the competition between the spin-orbit coupling and the Zeeman energy of the perpendicular field can be removed by a tilted field. In the case of $\theta\neq 0$, the energy levels can be calculated numerically. Using the expression in Eq. (4), we may make a truncation approximation by keeping the Landau levels with $n<N$ such that the dimensionality of matrix is reduced to $2N\times 2N$. Numerical diagonalization of the matrix can give us the energy eigenvalues.

Alternatively, the gap can also be calculated approximately by the degenerate perturbation theory. We take the partial Hamiltonian $H'=g_s\mu_B B_\perp\,\tan\,\theta\sigma_x/2$ as a perturbation, and express it in the subspace spanned by the two states $\Phi_{n,k,1}$ and $\Phi_{n+1,k,-1}$ near the resonant point,

$$\widetilde{H'} = \begin{bmatrix} 0 & i\Delta/2 \\ -i\Delta/2 & 0 \end{bmatrix}, \tag{6}$$

where the gap Δ is

$$\Delta = g_s\mu_B B_\perp\,\tan\,\theta\,\cos\,\theta_{n1}\,\sin\,\theta_{n+1,-1}. \tag{7}$$

In Fig. 1, we present the energy levels of $\theta=0$ as a function of $B_\perp$. The parameters used are $\lambda=9\times 10^{-12}$ eV m, $g_s=4$, and $m=0.05m_e$.[11] The arrow denotes a level crossing at $B_0\approx 2.4$ T. The inset shows the energy gap as a function of the tilt angle θ with $B\perp=B_0$. We notice that the numerical and analytical results are in good agreements.

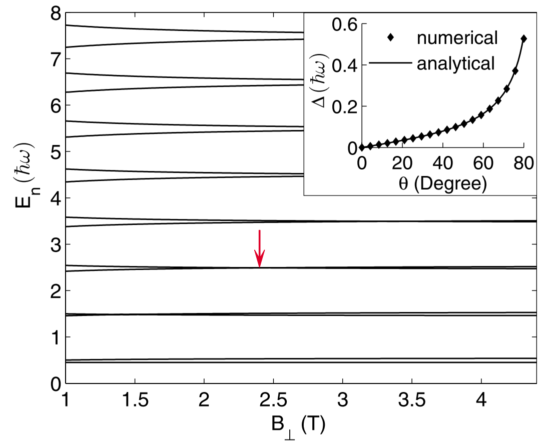

FIG. 1. (Color online) Energy levels as functions of the magnetic field when the tilt angle $\theta=0$, the arrow denotes a level-crossing point, which develops into a gap when the tilt angle increases. The inset shows the gap as a function of the tilt angle, reflects the accordance of the numerical calculation and the analytic expression Eq. (7). The energy has been scaled by $\hbar\omega=\hbar eB/m$.

B. Self-consistent Born approximation

In this section, we briefly review the general formalism of linear-response theory of SCBA for electron transport. We shall use this technique to investigate the transport properties of 2DEG with a Rashba coupling in a tilted magnetic field, especially near the resonant point. The effect of impurities will be taken into account in this formalism.

We consider a random configuration of impurities with short-range potentials $V(\mathbf{r})=\sum_{j=1}^{N_i} V\delta(\mathbf{r}-R_j)$, where $\mathbf{R}_j$ is the position of the jth impurity. The density of the impurities is $n_i=N_i/(L_xL_y)$. Generally speaking, the Green's functions for a specific configuration of random potential can be written as $\mathcal{G}^\pm(E)=[E-H_0-V(\mathbf{r})\pm i0^+]^{-1}$, where + and − correspond to the retarded and advanced Green's function, respectively. All transport quantities can be expressed in terms of the Green's function after averaging all possible configurations of the impurities. Using the conventional perturbation expansion with respect to $V(\mathbf{r})$, we can obtain the Dyson equation for the averaged Green's function $G^\pm$. The impurity effect is absorbed by a self-energy function $\Sigma^\pm$ as follows:

$$G^\pm(E) \equiv \langle \mathcal{G}^\pm(E)\rangle_c = [E - H_0 - \Sigma^\pm(E)], \tag{8}$$

where $\langle\cdots\rangle_c$ means the average over all the impurity configurations. In the SCBA, the self-energy operator can be expressed by $\Sigma^\pm=\langle VG^\pm(E)V\rangle_c$.[36–39] In the representation of the Landau levels, G, Σ, and V are expressed as matrices. For such a spin-independent impurity potential, previous works[36,40] proved that the self-energy is independent of n and k for a spin-independent Landau system. We find that the self-energies for a spin-orbit coupling system are independent of n and k,

$$\Sigma^\pm_{nk\sigma,n'k'\sigma'} = \delta_{nn'}\delta_{kk'}n_iV^2\frac{N_\phi}{L_xL_y}\sum_{n_1} G^\pm_{n_1\sigma,n_1\sigma'}. \tag{9}$$

Here we dropped the index k in $G^\pm$ because the averaged Green's functions are k independent.

C. Streda's formula for spin Hall conductivity

With the averaged Green's function in mind, we can use the Kubo formula to calculate the linear response of any physical quantity $\hat{O}$ to an external electric field E_{ext} in the ν direction,

$$\sigma_\nu^O = \lim_{E_{ext}\to 0} \langle \hat{O} \rangle_c / E_{ext}. \tag{10}$$

As the single-particle version of the Kubo formula, the Streda's formula is a conventional and powerful tool to study the transport property of 2DEG system under a magnetic field.[41] At the zero temperature, the formula is given by

$$\sigma_\nu^O(E_f) = ie\hbar \left\langle \int_{-\infty}^{Ef} dE \, \mathrm{Tr}\left[\hat{O} \frac{d\mathcal{G}^+(E)}{dE} v_\nu A(E) - \hat{O} A(E) v_\nu \frac{d\mathcal{G}^-(E)}{dE} \right] \right\rangle, \tag{11}$$

where $A \equiv (\mathcal{G}^- - \mathcal{G}^+)/(2\pi i)$ is the spectral function and $v_\nu \equiv \frac{1}{i\hbar}[r_\nu, H]$ is the velocity operator. The vertex correction has to be included because there are products of two Green's functions in the impurity average $\langle \cdots \rangle_c$. For a specific density of charge carriers, the Fermi energy as a function of the magnetic field is determined by

$$n_e = \left\langle \int_{-\infty}^{Ef} dE \, \mathrm{Tr}[A] \right\rangle_c .$$

Usually the Streda's formula is applied to calculate the electric conductance by replacing $\hat{O}$ by an electric current operator, $J_\nu = -e v_\nu$. In the present work, we intend to explore the spin transport in the system. The spin current is defined as $j_\mu^\alpha = (\hbar/4)\{v_\mu, \sigma_\alpha\}$, which is a tensor determined by both the motion direction of an electron and its polarization. In the framework of linear-response theory, the spin Hall conductivity $\sigma_{\mu\nu}^\alpha$, the ratio of the spin Hall current to an external field, can be calculated by substituting $\hat{O} = j_\mu^\alpha (\mu \neq \nu)$ in Eq. (11). The spin Hall conductivity comes from the contribution of all the electrons below the Fermi level. Opposite to those for the conductivity and Hall conductivity, it cannot be reduced to a Fermi edge quantity[36] because the spin current is not a commutator of any operator and the Hamiltonian.[42] For the purpose of our numerical calculation, we transform the Streda's formula into the following form:

$$\sigma_{\mu\nu}^\alpha(E_f) = \frac{e\hbar}{2\pi} \int_{-\infty}^{Ef} dE \, \mathrm{Tr}[j_\mu^\alpha (K_\nu^{+-} - K_\nu^{++} - K_\nu^{--})], \tag{12}$$

where

$$K_\nu^{+-} \equiv \frac{d\langle \mathcal{G}^+ v_\nu \mathcal{G}^- \rangle_c}{dE}, \tag{13}$$

$$K_\nu^{++} \equiv \left\langle \frac{d\mathcal{G}^+}{dE} v_\nu \mathcal{G}^+ \right\rangle_c, \quad K_\nu^{--} = [K_\nu^{++}]^+. \tag{14}$$

$K_\nu^{\sigma\sigma'}$ are determined in a set of Bethe-Salpeter-type equations,

$$K_\nu^{+-} = \frac{dG^+}{dE}[G^+]^{-1} F_\nu^{+-} + F_\nu^{+-}[G^-]^{-1} \frac{dG^-}{dE} + G^+ \langle V K_\nu^{+-} V \rangle_c G^-, \tag{15}$$

$$K_\nu^{++} = \frac{dG^+}{dE}[G^+]^{-1} F_\nu^{++} + G^+ \langle V K_\nu^{++} V \rangle_c G^+. \tag{16}$$

where $F_\nu^{\sigma\sigma'} = \langle \mathcal{G}^\sigma v_\nu \mathcal{G}^{\sigma'} \rangle_c$ $(\sigma, \sigma' \in \{+,-\})$ is the vertex operator, which satisfy the Bethe-Salpeter equation in the ladder approximation,

$$F_\nu^{\sigma\sigma'} = G^\sigma (v_\nu + \langle V F_\nu^{\sigma\sigma'} V \rangle_c) G^{\sigma'}. \tag{17}$$

These equations can be solved self-consistently and Eqs. (15) and (17) have multisolutions. From the continuity equation for charge current in equilibrium, we can derive an auxiliary equation,

$$\mathrm{Tr}[F_\nu^{+-}(\Sigma^- - \Sigma^+)] = 0. \tag{18}$$

Differentiating Eq. (18) with respect to E leads to another auxiliary equation,

$$\mathrm{Tr}[K_\nu^{+-}(\Sigma^- - \Sigma^+)] = -\mathrm{Tr}\left[F_\nu^{+-} \frac{d(\Sigma^- - \Sigma^+)}{dE} \right]. \tag{19}$$

dG^σ/dE is determined by an another self-consistent equation,

$$\frac{dG^\sigma}{dE} = -G^\sigma G^\sigma + G^\sigma \left\langle V \frac{dG^\sigma}{dE} V \right\rangle_c G^\sigma. \tag{20}$$

Since the density of states for each Landau level in SCBA has a semielliptic form,[43–45] it approaches zero and dG^σ/dE becomes infinity at the edge of each level. We find the integrand in Eq. (12) is always convergent in numerical calculation because the concurrence of $d\mathcal{G}^\sigma/dE$ and $\mathcal{G}^\sigma$ in Eqs. (13) and (14).

III. NUMERICAL RESULTS

Now we are ready to calculate the spin Hall conductivities numerically. In this paper, the electron density is fixed at $n_e = 2.9 \times 10^{15}$ m^{-2}. This value of n_e promises that the Fermi level is located near the resonant point with the filling factor $\nu = n_e/(N_\phi/L_x L_y) = 5$ while the magnetic field sweeps over the point, i.e., $B_\perp = B_0$ as indicated in Fig. 1. The other parameters are as the same as those used in Fig. 1. The density of impurities n_i and the strength of the impurity potential V are combined in one-parameter "scattering strength" $\Gamma \equiv n_i V^2 m/(2\pi\hbar^2)$. We assign Γ various values to investigate the impurity effect.

A. Disorder effect of resonant spin Hall effect

We first discuss the disorder effect of spin Hall conductance, especially near the crossing point. We apply the formula in Eq. (12) to calculate the spin Hall conductivity σ_{xy}^z around the resonant point $B_0 = 2.4$ T numerically for various strengths of disorder. Numerical results are plotted in Fig. 2. The dashed curve for $\Gamma = 0$ is from the solution in Ref. 12.

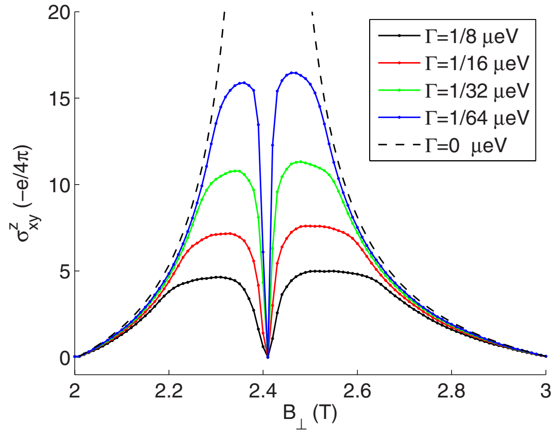

FIG. 2. (Color online) The spin Hall conductivity as a function of the magnetic field around the level-crossing point when $\theta=0$, for various scattering strength Γ.

The key feature of the disorder effect is the suppression of the spin Hall conductivity at the resonant point. The large spin Hall conductance exhibits when the field deviates from the resonant point and forms a double-peak structure. The weight of the spin Hall conductivity increases as the impurity strength decreases, which reflects the intrinsic properties of the resonance.

To understand the suppression of resonant spin Hall conductance, we plotted in Fig. 3 the distribution of the spin Hall conductivity on the electron's energy, $d\sigma_{xy}^z/dE$, around the crossing point. The total spin Hall conductivity $\sigma_{xy}^z(E_f) = \int_{-\infty}^{E_f}(d\sigma_{xy}^z/dE)dE$ are contributed by all the electron states under the Fermi level. It was observed that the energy levels are broadened due to the impurity scattering and the distribution of the spin Hall conductivity is inhomogeneous. So the magnitude and even the sign of the spin Hall conductivity can be varied by the Fermi level or the electron density. In the clean limit, a tiny external electric field can open a gap between the crossing levels, which leads to spin Hall conductance divergent. However, after the impurity scattering is taken into account, a tiny external field cannot open an energy gap any more because of the level broadening. As a result, the spin Hall conductance in a weak-field limit will be suppressed. However, once the external field becomes stronger than the level broadening, a large spin Hall conductance will appear. This can be seen from the case that the magnetic field deviates from the crossing point, i.e., the additional degeneracy of the two levels will be lifted and a strong spin Hall conductance recovers. This is the physical origin of the double-peak structure of the resonant spin Hall conductance. It is worth stressing that this suppression of resonant spin Hall conductance is different from the case in the absence of the Zeeman term. The Zeeman splitting may produce a nonzero spin Hall conductance in the Rashba system.[14]

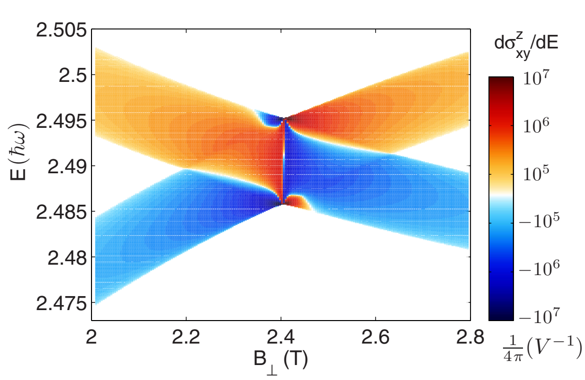

FIG. 3. (Color online) The distribution of the spin Hall conductivity on the electron's energy, $d\sigma_{xy}^z/dE$, as a function of the energy E and the magnetic field $B_\perp$ when the tilt angle $\theta=0$, with the impurity strength $\Gamma=1/32$ μeV. The energy E has been scaled by $\hbar\omega$.

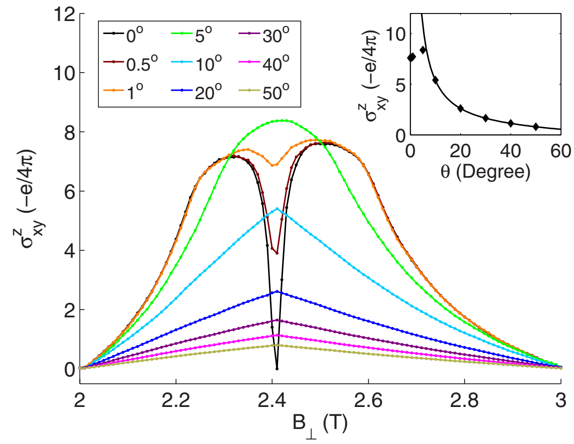

FIG. 4. (Color online) Spin Hall conductivity as a function of the perpendicular component of the magnetic field for various tilt angles θ. The inset compares the peak height of the spin Hall conductivity as function of θ from the numerical calculation (diamond) and from the analytical formula Eq. (21) (solid line). The scattering strength $\Gamma=1/16$ μeV.

B. Effect of a tilted field

To further illustrate the formation of the resonant spin Hall effect, we investigate the effect of the tilted magnetic field. Figure 4 shows the dependence of the spin Hall conductivity on the tilt angle near the resonant point. For the purpose of numerical calculation, we take the scattering strength $\Gamma=1/16$ μeV. As the tilted angle increases, the spin Hall conductivity at the resonant point will increase very quickly and the two peaks finally integrate into one. After that point, the spin Hall conductivity begins to decrease. These behaviors can be understood as the competition between the disorder broadening of the energy levels and the degeneracy lifting by the tilted field.

In the clean limit, the tilted field will remove the degeneracy of the energy crossing levels as shown in Fig. 1. We can estimate the peak height of the spin Hall conductivity as a function of the the tilt angle by a perturbation calculation adopted in Sec. II A. Diagonalizing the truncated two-level Hamiltonian in Eq. (6), we get the modified eigenstates $\Psi_\pm = (\Phi_{nk1} \pm i\Phi_{n+1,k,-1})/\sqrt{2}$, and the energy correction $E_\pm = \pm\Delta/2$ with Eq. (7). If the Fermi level just lies between the energy levels, the spin Hall conductivity is mainly attributed to Ψ_-, which can be calculated by the Kubo formula,[12,13]

$$\sigma_{xy}^{z(1)}(B_0) = \sum_k \left(\frac{\langle \Psi_+ | ey | \Psi_- \rangle \langle \Psi_- | j_x^z | \Psi_+ \rangle}{E_- - E_+} + \text{c.c} \right)$$
$$= \frac{\hbar e^2 B_0}{4\pi m \Delta}[(n+1)\cos^2\theta_{n1}\cos^2\theta_{n+1,-1} - n\sin^2\theta_{n1}\sin^2\theta_{n+1,-1}], \quad (21)$$

where $y=(mv_x|_{\lambda=0}-\hbar k)/eB$.

The impurity scattering will cause the level broadening. If the level broadening is larger than the gap caused by the tilted field, the impurity effect is dominant. Otherwise the tilted field effect will be dominant. The level broadening is characterized by the half width of the semielliptic density of states of the Landau levels, which can be estimated approximately by[45] $\gamma=2\sqrt{n_i V^2 N_\Phi / L_x L_y}$. When $\Delta > 2\gamma$, the effect of the tilted field turns out to be dominant. The inset of Fig. 4 presents the data of the peak height of the spin Hall conductivity as a function of θ from the numerical calculation (diamond) and from the analytic formula (solid line) in Eq. (21), They are in a good agreement when $\theta \gtrsim 10°$, which is very close to the estimated value $\theta \approx 8°$ from $\hbar\omega$.

IV. SUMMARY

In summary, we applied the Streda's formula to study the disorder effect of the resonant spin Hall effect in the 2DEG system with the Rashba interaction in a tilted magnetic field. Considering the vertex corrections in the self-energy, we find that the main effect of the impurity scattering is to broaden the Landau levels. In the framework of linear response, the electric field is taken to approach zero, and the energy splitting caused by the electric field is always less than the broadening of the Landau levels. Thus a tiny external field cannot remove the additional degeneracy of the energy levels at the resonant point. As a result, the spin Hall conductance will be suppressed at the point. When the magnetic field slightly deviates the resonant point or a tilted field is applied, the degeneracy will be removed, a large spin Hall conductance will be recovered. The spin Hall conductance exhibits a double peak around the resonant point. From the effect of a tilted field, we believe that a finite electric field, if it is strong enough to overcome the energy-level broadening, will recover the spin Hall effect even at the resonant point. This is quite different from the disorder effect of spin Hall effect in the Rashba system in the absence of magnetic field.[18]

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